Variational approach to solitons in systems with cascaded $\chi^{(2)}$ nonlinearity

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Soliton propagation in media with cascaded $\chi^{(2)}$ nonlinearities is investigated. The exact soliton solutions of the governing equations are studied. They are either independent of any free parameters or depend only on one. In the limit of a large mismatch the equations have a well known two-parameter nonlinear Schrödinger soliton solution. The analogous two-parameter families of solutions for arbitrary phase mismatch are found by means of the variational approach. This proves Menyuk's robustness hypothesis. [S1063-651X(97)10701-2]

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I. INTRODUCTION

The cascading of the second-order nonlinearities $\chi^{(2)}$: $\chi^{(2)}$ has attracted a special interest because of its effective contribution to the third order nonlinearities $\chi^{(3)}$, and been proved to offer a promising approach to obtaining an efficient third-order nonlinear optical response [1–5]. It is well known that wave propagation in materials with effective $\chi^{(3)}$ can be described by a nonlinear Schrödinger (NLS) equation, which has exact soliton solutions [6]. The equations governing the wave propagation in media with a cascaded $\chi^{(2)}$ nonlinearity were shown to reduce to the NLS equation in the limit of large phase mismatch and negligible conversion into the second harmonic [2,3]. In addition, the governing equations also have a number of exact soliton solutions for arbitrary mismatch [7,4,5,8,9]. Detailed analyses of the exact soliton solutions have been carried out in a recent paper [9]. It should be pointed out here that analogous equations also appear in the theory of Fermi-resonance soliton propagation along the interface of organic semiconductors [8,10,11]. The stability of these exact solutions was studied in [12]. Exact soliton solutions of the equations governing the wave propagation in media with cascaded $\chi^{(2)}$ nonlinearity are either independent of any free parameters or depend only on one parameter. In contrast, asymptotically exact NLS soliton solutions depend on two free parameters. This lies in the basis of Menuyk's robustness hypothesis of solitons [13], which states that analogous two-parameter excitations should exist for arbitrary mismatch. In what follows we investigate the robustness hypothesis by means of a variational approach [14] to the wave propagation in media with cascaded $\chi^{(2)}$ nonlinearities. Note that such an approach has been used in the recent paper [15] with the different choice of trial functions. Our choice appears more natural since it enables us to reproduce the exact solutions for the appropriate values of the corresponding parameters.

The derivation of the equations governing the propagation of fundamental and second-harmonic waves in a medium with cascaded $\chi^{(2)}$ nonlinearities is described in detail in [9]. In the normalized form they can be written as

$$
\begin{aligned}
&i\,\frac{\partial B}{\partial \xi}-\gamma_b\,\frac{\partial^2 B}{\partial s^2}+2B^*C=0,\\
&i\,\frac{\partial C}{\partial \xi}-\beta C-\delta\,\frac{\partial C}{\partial s}-\gamma_c\,\frac{\partial^2 C}{\partial s^2}+B^2=0.
\end{aligned}
\tag{1}
$$

In Eqs. (1), B and C are the normalized slowly varying envelopes of the fundamental and second-harmonic waves, ξ is the normalized coordinate in the direction of wave propagation, and β is the normalized phase mismatch. The coordinate s can be treated either as the time coordinate in the problem of wave propagation in a medium with dispersion, or a transverse coordinate in the problem of wave propagation in a planar waveguide; the coefficients γ_b, γ_c, and δ characterize the wave evolution in the medium. For definiteness we consider below the case of a planar waveguide which seems to be more realistic for an observation of solitary waves.

Exact solutions of equations (1) provide the starting point for the choice of the trial functions, so at first we shall discuss these solutions.

II. EXACT SOLITON SOLUTIONS

Several families of exact soliton solutions were found in [8–10]. In what follows we extend the method of [8,10], restricting our consideration to the case $\delta=0$.

Let us seek the solutions of Eqs. (1) in the form

$$
\begin{aligned}
&B=F\ \exp\{(-ik\xi+i\omega s)/2\},\\
&C=\mu F\ \exp\{-ik\xi+i\omega s\},\quad F=F(s-v\xi),
\end{aligned}
\tag{2}
$$

where μ is a constant. Substitution of Eqs. (2) into Eqs. (1) gives

$$
\begin{aligned}
&\left(\frac{k}{2}+\frac{\gamma_b\omega^2}{4}\right)F-i(v+\gamma_b\omega)F'-\gamma_bF''-2F^2\mu=0,\\
&(k-\beta+\gamma_c\omega^2)F-i(v+2\gamma_c\omega)F'-\gamma_cF''-F^2/\mu=0.
\end{aligned}
\tag{3}
$$

The imaginary parts of these equations vanish when

$$\text{(a)} \quad \omega=0, \quad v=0,$$
$$\text{(b)} \quad \gamma_b=2\gamma_c, \quad v=-\gamma_b\omega=-2\gamma_c\omega. \tag{4}$$

Let us first consider case (a). Equations (3) for function F are compatible if

$$\frac{k/2}{k-\beta}=\frac{\gamma_b}{\gamma_c}=2\mu^2, \tag{5}$$

which determines μ and k:

$$\mu=\pm\left(\frac{\gamma_b}{2\gamma_c}\right)^{1/2}, \quad k=\frac{2\beta\gamma_b}{2\gamma_b-\gamma_c}. \tag{6}$$

Under these conditions, F satisfies the equation

$$F''-\frac{\beta}{2\gamma_b-\gamma_c}F\pm\left(\frac{2}{\gamma_b\gamma_c}\right)^{1/2}F^2=0. \tag{7}$$

Its first integral has the form

$$\left(\frac{dF}{dx}\right)^2=(\alpha\mp F)F^2-(\alpha\mp\gamma)\gamma^2, \tag{8}$$

where

$$x=\left(\frac{2\sqrt{2}}{3\sqrt{\gamma_b\gamma_c}}\right)^{1/2}s, \quad \alpha=\frac{3\sqrt{\gamma_b\gamma_c}}{2\sqrt{2}}\frac{\beta}{\gamma_c-2\gamma_b}, \tag{9}$$

and γ is an integration constant. Zeros of the polynomial on the right hand side of Eq. (8) are

$$F_1=\gamma, \quad F_2=\tfrac{1}{2}[\alpha-\gamma\pm\sqrt{(\alpha+\gamma)^2-4\gamma^2}] \tag{10}$$

for the upper sign choice, and

$$F_1=\gamma, \quad F_2=\tfrac{1}{2}[-\alpha-\gamma\pm\sqrt{(\alpha-\gamma)^2-4\gamma^2}], \tag{11}$$

for the lower sign choice. Their locations on the F axis depend on the values of α and γ. There are different cases in which F oscillates between the zeros where this polynomial is positive. In all these cases the solution of Eq. (8) can be expressed in terms of elliptic functions. However, when the two zeros of the polynomial, between which it is negative, coincide, we obtain the soliton solutions. Simple analysis (see details in [8]) leads to the ''bright'' soliton solution (i.e., higher intensity pulse on the zero background)

$$B=\frac{\alpha\ \mathrm{sgn}(\mu)e^{-ik\xi/2}}{\cosh^2\kappa s}, \quad C=\frac{\alpha|\mu|e^{-ik\xi}}{\cosh^2\kappa s}, \tag{12}$$

and the ''dark'' soliton solution (i.e., lower intensity pulse on the constant nonzero background)

$$B=-\alpha\ \mathrm{sgn}(\mu)\left(\frac{1}{\cosh^2\kappa s}-\frac{2}{3}\right)e^{-ik\xi/2},$$
$$C=-\alpha|\mu|\left(\frac{1}{\cosh^2\kappa s}-\frac{2}{3}\right)e^{-ik\xi}, \tag{13}$$

where

$$\kappa=\frac{1}{2}\left(\frac{\beta}{2\gamma_b-\gamma_c}\right)^{1/2}, \quad \frac{\beta}{2\gamma_b-\gamma_c}>0, \tag{14}$$

for the ''bright'' soliton, and

$$\kappa=\frac{1}{2}\left(\frac{\beta}{\gamma_c-2\gamma_b}\right)^{1/2}, \quad \frac{\beta}{\gamma_c-2\gamma_b}>0 \tag{15}$$

for the ''dark'' soliton, while μ, k, and α are given by Eqs. (6) and (9), respectively. These solutions do not have any free parameters.

In the same way a particular one-parameter solution can be obtained in case (b) [see Eqs. (4)]. In this case we have

$$\mu=\pm 1, \quad k=\tfrac{4}{3}\beta-\frac{\gamma_b}{2}\omega^2, \tag{16}$$

and integration of the Eq. (7) for the function F leads to the bright soliton

$$B=\frac{\alpha\mu\ \exp(-ik\xi/2+i\omega s/2)}{\cosh^2[\kappa(s-v\xi)]}, \quad C=\frac{\alpha\ \exp(-ik\xi+i\omega s)}{\cosh^2[\kappa(s-v\xi)]}, \tag{17}$$

and the dark soliton

$$B=-\alpha\mu\left(\frac{1}{\cosh^2[\kappa(s-v\xi)]}-\frac{2}{3}\right)\exp(-ik\xi/2+i\omega s/2),$$
$$C=-\alpha\left(\frac{1}{\cosh^2[\kappa(s-v\xi)]}-\frac{2}{3}\right)\exp(-ik\xi+i\omega s), \tag{18}$$

where

$$\alpha=\frac{\beta}{2}, \quad \kappa=\left(\left|\frac{\beta}{2\gamma_c}\right|\right)^{1/2}, \tag{19}$$

and $\beta/2\gamma_c>0$ for the bright soliton and $\beta/2\gamma_c<0$ for the dark soliton.

Note that, as v approaches zero, we return to solutions (12) and (13) with $\gamma_b=2\gamma_c$. The solutions obtained depend on one free parameter ω. This kind of solution has been discussed in [8,9].

It is well known that there is another possibility to find an asymptotically exact solution of Eqs. (1). As many authors have shown [2,3], in the limit of large mismatch β the wave propagation is approximately described by the nonlinear Schrödinger equation. Indeed, when $\beta\gg|\gamma_c|$, $|C|\ll B^2$, from the second equation of Eqs. (1) we obtain

$$C\approx-\frac{1}{\beta}B^2. \tag{20}$$

Substitution of Eq. (19) into the first equation of Eqs. (1) yields

$$i\frac{\partial B}{\partial\xi}-\gamma_b\frac{\partial^2 B}{\partial s^2}+\frac{2}{\beta}|B|^2B=0. \tag{21}$$

This NLS equation has the well-known soliton solution

$$B=\sqrt{-\beta\gamma_c}\frac{\kappa\ \exp[-i(k\xi-\omega s)/2]}{\cosh[\kappa(s-v\xi)]}, \tag{22}$$

which depends on two free parameters κ and ω, while v and k are given by

$$v=-\gamma_b\omega, \quad k=2\gamma_b(\kappa^2-\omega^2/4). \tag{23}$$

The expression for the field C follows from Eq. (20).

Thus there exist several families of exact or asymptotically exact solutions of Eqs. (1). Solution (12) does not contain any free parameters. Solution (17) depends on one free parameter ω, and exists if only $\gamma_b=2\gamma_c$. Finally, solution (22) depends on two free parameters κ and ω but is exact asymptotically for large values of β only. One may expect that in the system under consideration the analogous solitonic solutions should exist which belong to a two-parameter family, and are not under constraint of any conditions. This possibility will be discussed in Secs. III and IV with the use of the variational approach. In what follows we confine ourselves to the case of bright soliton solutions only. As we have seen, there are two families of the exact solutions with different shapes of solitons. Consequently, there should be two families of variational solutions and they will be considered separately.

III. VARIATIONAL APPROACH TO CASCADED $\chi^{(2)}$ SOLITONS: THE FIRST FAMILY OF SOLUTIONS

The variational approach is based on the possibility to present Eqs. (1) as Lagrange equations corresponding to the Lagrangian

$$L=\int_{-\infty}^{\infty}\left[\frac{i}{2}(B_\xi^*B-B^*B_\xi+C_\xi^*C-C^*C_\xi)+\frac{i\delta}{2}(C^*C_s-C_s^*C)\right]ds+\int_{-\infty}^{\infty}[\beta C^*C-\gamma_bB_s^*B_s-\gamma_cC_s^*C_s+B^2C^*+B^{*2}C]ds, \tag{24}$$

where $B_\xi=\partial B/\partial\xi$, $B_s=\partial B/\partial s$, and so on.

Solution (17) suggests the following form for the trial functions

$$B=\frac{b\,\exp(i\varphi/2)}{\cosh^2[\kappa(s-\zeta)]}, \quad C=\frac{c\,\exp(i\varphi)}{\cosh^2[\kappa(s-\zeta)]}, \tag{25}$$

where

$$\varphi=\omega(s-\xi/2)+\eta. \tag{26}$$

Substitution of Eqs. (25) into Eq. (24) yields

$$L=\frac{4b^2}{3\kappa}\left[-\gamma_b\left(\frac{\omega^2}{4}+\frac{4}{5}\kappa^2\right)-\frac{1}{2}\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\frac{1}{4}\zeta\omega_\xi\right]+\frac{4c^2}{3\kappa}\left[\beta-\gamma_c\left(\omega^2+\frac{4}{5}\kappa^2\right)-\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\frac{1}{2}\zeta\omega_\xi-\delta\omega\right]+\frac{32}{15}\frac{b^2c}{\kappa}. \tag{27}$$

The Lagrange equations for the variables b, c, η, ζ, ω, and κ are

$$-\gamma_b\left(\frac{\omega^2}{4}+\frac{4}{5}\kappa^2\right)-\frac{1}{2}\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\tfrac{1}{4}\zeta\omega_\xi+\tfrac{8}{5}c=0, \tag{28}$$

$$\beta-\gamma_c\left(\omega^2+\frac{4}{5}\kappa^2\right)-\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\tfrac{1}{2}\zeta\omega_\xi-\delta\omega++\frac{4}{5}\frac{b^2}{c}=0, \tag{29}$$

$$\frac{d}{d\xi}\left(\frac{b^2+2c^2}{\kappa}\right)=0, \tag{30}$$

$$\frac{\omega}{2}\left[\frac{d}{d\xi}\left(\frac{b^2+2c^2}{\kappa}\right)\right]+\frac{b^2+2c^2}{\kappa}\frac{d\omega}{d\xi}=0, \tag{31}$$

$$\frac{d}{d\xi}\left(\frac{b^2+2c^2}{\kappa}\zeta\right)+\frac{b^2}{\kappa}(\zeta_\xi+2\omega\gamma_b)+\frac{2c^2}{\kappa}(\zeta_\xi+4\omega\gamma_c+2\delta)=0, \tag{32}$$

$$b^2\left[-\gamma_b\left(\frac{\omega^2}{4}+\frac{4}{5}\kappa^2\right)-\frac{1}{2}\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\frac{1}{4}\zeta\omega_\xi\right]+\frac{8}{5}\gamma_bb^2\kappa^2+\frac{8}{5}\gamma_cc^2\kappa^2+c^2\left[\beta-\gamma_c\left(\omega^2+\frac{4}{5}\kappa^2\right)-\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\frac{1}{2}\zeta\omega_\xi-\delta\omega\right]+\frac{8}{5}b^2c=0. \tag{33}$$

From Eqs. (30) and (31) we obtain $d\omega/d\xi=0$, i.e., ω is a constant. Then the differentiation of the phase φ [Eq. (26)] with respect to ξ yields the expression for the ''wave number'' k,

$$k=\frac{\omega}{2}\zeta_\xi-\eta_\xi. \tag{34}$$

From Eq. (32) we obtain

$$v=\zeta_\xi=-\omega\frac{\gamma_bb^2+4\gamma_cc^2}{b^2+2c^2}, \tag{35}$$

and, consequently,

$$k=-\eta_\xi-\frac{\omega^2}{2}\frac{\gamma_bb^2+4\gamma_cc^2}{b^2+2c^2}. \tag{36}$$

Substitution of Eqs. (28) and (29) into Eq. (33) yields the relation

$$2(\gamma_bb^2+\gamma_cc^2)\kappa^2=b^2c, \tag{37}$$

so Eqs. (28) and (29) can be written in the form

$$c=\frac{5}{8}\left[\frac{k}{2}+\gamma_b\left(\frac{\omega^2}{4}+\frac{4}{5}\kappa^2\right)\right], \tag{38}$$

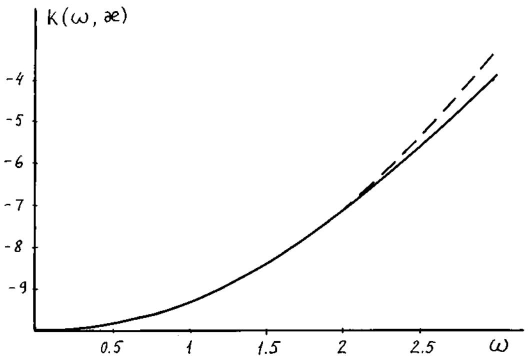

FIG. 1. The dependence of the wave number of the solitonic excitation on ω for the first family of solutions. The lower curve corresponds to the exact variational solution (40), and the upper curve to its approximation (41). The chosen parameter values are equal to $\gamma_b=\gamma_c=-1$, $\beta=-5$, and $\delta=0$. The soliton width κ is calculated with the use of formula (14).

$$b^2=\frac{25}{32}\left[k-\beta+\gamma_c\left(\omega^2+\frac{4}{5}\kappa^2\right)-\delta\omega\right]\times\left[\frac{k}{2}+\gamma_b\left(\frac{\omega^2}{4}+\frac{4}{5}\kappa^2\right)\right]. \qquad (39)$$

When we substitute these expressions into Eq. (37) we obtain the equation for k. The solution of the equation obtained gives the wave number k as a function of two parameters ω and κ:

$$k(\omega,\kappa)=\frac{\beta-\delta\omega}{2}+\frac{2}{5}(6\gamma_b+\gamma_c)\kappa^2-\frac{1}{4}(\gamma_b+2\gamma_c)\omega^2 \pm\left(\left[\frac{\beta-\delta\omega}{2}-\frac{2}{5}(6\gamma_b-\gamma_c)\kappa^2+\frac{1}{4}(\gamma_b-2\gamma_c)\omega^2\right]^2+\frac{256}{25}\gamma_b\gamma_c\kappa^4\right)^{1/2}. \qquad (40)$$

The choice of the sign before the square root depends on the sign of the expression in the square brackets, and should be made so that Eq. (40) reproduces the exact solution given by Eq. (12) when $\omega=0$ and κ is given by Eq. (14). It is worthwhile to present a simple and useful formula keeping only the first two terms in the series expansion of Eq. (40) in powers of ω^2 when κ is given by Eq. (14),

$$k(\omega)\simeq\frac{2\beta\gamma_b}{2\gamma_b-\gamma_c}-\frac{3\gamma_b\gamma_c}{2(\gamma_b+\gamma_c)}\omega^2. \qquad (41)$$

This reproduces the exact solutions (6) and (16) in both specific cases $\omega=0$ and $\gamma_b=2\gamma_c$. In Fig. 1 a plot of the function $k(\omega)$ is shown. The lower line shows the exact variational solution (40) and the upper line shows its approximation (41).

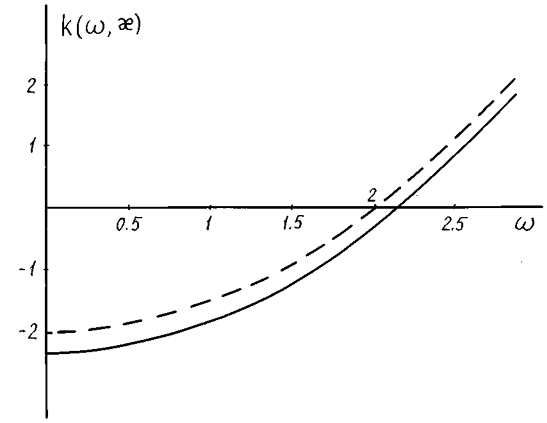

FIG. 2. The dependence of the wave number of the solitonic excitation on the wave number for the second family of solutions. The lower curve corresponds to the exact variational solution (49), and the upper curve to its approximation (23). The chosen values of the parameters are equal to $\gamma_b=\gamma_c=-1$, $\beta=10$, $\delta=0$, and $\kappa=1$. The relative difference between them is of order $\gamma_b\gamma_c\kappa^4/\beta^2$.

IV. VARIATIONAL APPROACH TO CASCADED $\chi^{(2)}$ SOLITONS: THE SECOND FAMILY OF SOLUTIONS

In this section we start from the NLS asymptotically exact solution (22), which suggests the following form for the trial functions:

$$B=\frac{b\,\exp(i\varphi/2)}{\cosh[\kappa(s-\zeta)]},\quad C=\frac{c\,\exp(i\varphi)}{\cosh^2[\kappa(s-\zeta)]}, \qquad (42)$$

where φ is given by Eq. (26). These trial functions differ from the functions given by Eq. (25) only by the power of $\cosh[\kappa(s-\zeta)]$ in the expression for B. Their substitution into the Lagrangian yields

$$L=\frac{2b^2}{\kappa}\left[-\gamma_b\left(\frac{\omega^2}{4}+\frac{\kappa^2}{3}\right)-\frac{1}{2}\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\frac{1}{4}\zeta\omega_\xi\right]+\frac{4c^2}{3\kappa}\left[\beta-\gamma_c\left(\omega^2+\frac{4}{5}\kappa^2\right)-\left(\frac{\omega}{2}\zeta_\xi-\eta_\xi\right)+\frac{1}{2}\zeta\omega_\xi-\delta\omega\right]+\frac{8}{3}\frac{b^2c}{\kappa}. \qquad (43)$$

The Lagrange equations are similar to those given by Eqs. (28)–(33). We obtain again $\omega=$const, so that k is given by Eq. (34). In the case under consideration we obtain

$$v=\zeta_\xi=-\omega\,\frac{\gamma_b b^2+\frac{8}{3}{}_c c^2}{b^2+\frac{4}{3}c^2}, \qquad (44)$$

and, hence,

$$k=-\eta_\xi-\frac{\gamma_b b^2+\frac{8}{3}\gamma_c c^2}{b^2+\frac{4}{3}c^2}. \qquad (45)$$

Instead of Eqs. (37)–(39), we obtain

$$\left(\gamma_b b^2+\frac{8}{5}\gamma_c c^2\right)\kappa^2=b^2c, \tag{46}$$

$$c=\frac{3}{4}\left[\frac{k}{2}+\gamma_b\left(\frac{\omega^2}{4}+\frac{\kappa^2}{3}\right)\right], \tag{47}$$

$$b^2=\frac{3}{4}\left[k-\beta+\gamma_c\left(\omega^2+\frac{4}{5}\kappa^2\right)+\delta\omega\right]\left[\frac{k}{2}+\gamma_b\left(\frac{\omega^2}{4}+\frac{\kappa^2}{3}\right)\right]. \tag{48}$$

Substitution of the last two equations into Eq. (46) yields the equation for k. Its solution has the form

$$k(\omega,\kappa)=\frac{\beta-\delta\omega}{2}+\left(\gamma_b+\frac{2}{5}\gamma_c\right)\kappa^2-\frac{1}{4}(\gamma_b+2\gamma_c)\omega^2 \pm\left\{\left[\frac{\beta-\delta\omega}{2}-\left(\gamma_b-\frac{2}{5}\gamma_c\right)\kappa^2+\frac{1}{4}(\gamma_b-2\gamma_c)\omega^2\right]^2 +\frac{64}{15}\gamma_b\gamma_c\kappa^4\right\}^{1/2}. \tag{49}$$

The sign before the square root depends on the sign of the expression in the square brackets, and should be chosen so that Eq. (49) reproduced Eq. (23) for $|\beta|\gg|\gamma_b|,|\gamma_c|$. In Fig. 2 the plots of the function $k(\omega)$ are shown for κ given by Eq. (14). The lower line corresponds to the exact variational solution (49), and the upper line to its asymptotic form (22). It is seen that the relative difference between the solutions is of the order of $\gamma_b\gamma_c\kappa^4/\beta^2\ll 1$, as it should be expected.

V. CONCLUSION

In this paper we investigated the soliton propagation in media with cascaded $\chi^{(2)}$ nonlinearities. We studied the exact soliton solutions of the equations governing the wave propagation in the media under consideration. The exact solutions discussed are either independent of any free parameters or depend only on one free parameter. In the framework of variational approach we found soliton solutions for arbitrary phase mismatch which depend on two free parameters. In the limiting cases the solutions obtained reduce to the exact or asymptotically exact solutions of the governing equations.

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